MMath II - Mid Semestral Examination - Topology III

Time : 2 Hours Max. Marks : 40

Answer all questions. Give complete justifications.

(1) Decide whether the following statements are True or False. Give complete justifications. .

(a) If E is the universal cover of $S^1 \vee S^2$, then $H^2(E;\mathbb{Z})$ is not finitely generated.

(b) There exist spaces X, Y and a map $f : X \longrightarrow Y$ such that $f^* : H^1(Y;\mathbb{Z}) \longrightarrow H^1(X;\mathbb{Z})$ is $1-1$ but not onto and $f^* : H^4(Y;\mathbb{Z}) \longrightarrow H^4(X;\mathbb{Z})$ is onto but not $1-1$.

(c) Let $X = D^3 \cup_f S^2$ where $f : S^2 \longrightarrow S^2$ is a map of degree 3. Then the quotient map

$$q : X \longrightarrow X/S^2$$

is essential.

(d) There does not exist a space X with $H^1(X;\mathbb{Z}) \cong \mathbb{Z} \oplus \mathbb{Z}_2$.

(e) The subspace $\mathbb{R}P^2$ of $\mathbb{R}P^3$ is a deformation retract of an open set in $\mathbb{R}P^3$. $[4 \times 5 = 20]$

(2) Let $\mathcal{C}$ be a chain complex of free abelian groups. Show that if $H_i(\mathcal{C})$ is finitely genrated for all i, then

$$H_i(\mathcal{C};\mathbb{Z}_n) \cong H^i(\mathcal{C};\mathbb{Z}_n)$$

for all i. Is the isomorphism natural with respect to chain maps? [6+2]

(3) Compute the groups $H_i(X;G)$ and $H^i(X;G)$ where $X = S^2 \times \mathbb{R}P^2$ and $G = \mathbb{Z}, \mathbb{Z}_2$. [12]